

Activity report. Obtained results

Project: 47/2011. Project Code: PN-II-ID-PCE-2011-3-0195

Project Title: QUALITATIVE AND NUMERICAL ANALYSIS OF NONLINEAR PROBLEMS ON FRACTALS

Project Director: Prof. PhD. Vicentiu Radulescu, Department of Mathematics, University of Craiova

Stage 1. Year 2011

I. Obtained Results

In this project, in the mentioned period of 2011, the following results were obtained, in accordance with the objectives, which were entirely fulfilled:

A. Articles accepted for publication

M. Boureau, V. Radulescu, Anisotropic Neumann problems in Sobolev spaces with variable exponent, *Nonlinear Analysis: Theory, Methods and Applications* (Elsevier, 2010 ISI Impact Factor: 1.279. Relative Ranking within Categories: Mathematics: 26/277; Mathematics, Applied: 50/236), in press (doi:10.1016/j.na.2011.09.033).

The article can be found in section Articles in Press at <http://www.sciencedirect.com/science/article/pii/S0362546X1100664X>

B. Articles accepted to be published

G. Bonanno, G. Molica Bisci, V. Radulescu, Variational analysis for a nonlinear elliptic problem on the Sierpinski gasket, *ESAIM: Control, Optimisation and Calculus of Variations* (Cambridge University Press, 2010 ISI Impact Factor: 1,221. Relative Ranking within Categories: Automation & Control Systems: 27/60; Mathematics, Applied: 55/236), accepted to be published.

This paper will be reported next year under this grant. The reason is that the article was accepted for publication recently, until the time of writing this report the paper has not yet received a Digital Object Identifier (DOI).

II. Conferences, lectures at scientific seminars

Proprietati calitative ale solutiilor unor probleme de valori proprii neliniare, Scientific Seminar of the Department of Mathematics, University Ovidius Constanta, 21.10.2011 (http://math.univ-ovidius.ro/Doc/Cercetare/SeminarFiles/CMAT/2011/SSDMAT_Abstract_20111021.pdf).

Bifurcation phenomena associated to degenerate or singular elliptic equations, Oxford Partial Differential Equations Seminar (<http://www.maths.ox.ac.uk/node/16632>), University of Oxford, 14.11.2011.

Both exposures were made on topics related to the theme of this grant. Oxford PDE seminar is organised by Professor Sir John Ball and is one of the most prestigious seminars on this field in the world. At the invitation to speak at this seminar the director of this research grant held discussions with many colleagues in England on the basis of modern research directions to be addressed in the grant.

III. Detailing results

M. Boureanu, V. Radulescu, Anisotropic Neumann problems in Sobolev spaces with variable exponent, *Nonlinear Analysis: Theory, Methods and Applications* (Elsevier, 2010 ISI Impact Factor: 1.279. Relative Ranking within Categories: Mathematics: 26/277; Mathematics, Applied: 50/236), in press (doi:10.1016/j.na.2011.09.033)

The paper fits into the theme and objectives of the grant and is accepted for publication in one of the most prestigious nonlinear analysis journals with a very good visibility in the international scientific community. The paper treats a stationary nonlinear boundary value problem. One of its main strengths is the presence of a homogeneous differential operator with a complicated structure, which began to be studied very recently. This requires the use of a suitable functional framework, corresponding to anisotropic Orlicz-Sobolev spaces which characterise nonhomogeneous environments. Issues such as these are applicable in non-Newtonian fluid theory (electrorheological) in image processing and robotics. Operators used in this paper extend in a very general framework the operator of Laplace and the p -Laplacian operator. Another technical difficulty of this work is related to using a boundary conditions of nonlinear Neumann type, which corresponds much better to models in mathematical physics.

The paper first obtains a compact dive for the spaces with variable exponent used in the article. This property extends Rellich Kondrachov-type theorems from the context of usual Sobolev type spaces. The main result of the paper establishes a qualitative result and conditions for the existence of an weak solution. Demonstration techniques are variational and are based on associated energetic coercivity functionality and the critical point results or estimates elliptical. The method introduced in the proof of this result can be extended to treat large class of nonlinear elliptic problems. More reviews completes this work, especially related to important assumptions used in the main result of the paper.

Other results obtained in this year (some are in press at important publications) refer to the development of variational analysis techniques in the context of nonlinear problems on fractals, for example Sierpinski's carpet. Different results are obtained for the existence of solution in non-standard cases. We intend to continue these results, and approach singular phenomena such as the existence of singular solutions with exhibitors at the boundary in the case of open fractal. Were

obtained more preliminary results in this case. Some are included in articles submitted for publication and are in the process of review, other articles are subject to editing phase. After the discussions of the Director of this grant with several experts on the subject in the UK, Italy, France and Slovenia emerged the main directions which will be addressed:

- Study solutions with explosion at the boundary in the domains with irregular border (fractal type) necessary and sufficient conditions for the existence of such singular solutions, results of uniqueness, asymptotic behaviour of solutions (speed burst at the border). In this study at the moment the aim is the premiere use of a theory that belongs to the grant director in the domains with smooth boundary, respectively Karam's theory of functions with regular variation (or fast) to infinity. The study of these phenomena is related to the logistics equation, related to important phenomena of population dynamics. A comprehensive approach to these phenomena in the case of nonfractal domains can be found in the recent monograph: M. Ghergu, V. Radulescu, *Nonlinear PDEs: Mathematical Models in Biology, Chemistry and Population Genetics*, Springer Monographs in Mathematics, Springer Verlag, Heidelberg, 2011.
- Study degenerate and singular phenomena in the case of nonlinear equations on fractals. We propose the study of Lane-Emden and Emden-Fowler type problems with singular nonlinearity on fractals and the rigorous approach to the bifurcation problems associated. Such problems haven't been studied to date and are of great importance in connection with phenomena of type *boundary-layer* for viscous fluids, plasma or laser beam theory models of homogeneous chemical catalysts. In this framework we aim not only to obtain qualitative results but also to study the competition between nonlinear terms and rugged border of the field, here appears to be new phenomena, for example the impossibility of flow conditions (described by Neumann boundary conditions). Equally we are interested in obtaining numerical results, which validate the theoretical results obtained.

In all cases we are interested in the use of modern mathematical apparatus for the study of mathematical models on fractals: variational principles, techniques of perturbation, asymptotic analysis, numerical methods. B. Mandelbrot is the founder of fractal theory (late 60s) and the article that drew attention to the strength of this theory is *How Long is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension*, Science 156, No. 3775 (1967), 636-638. In all the results that we intend to obtain in this grant we consider highlighting new qualitatively phenomena at the interface between modern fractal theory and nonlinear analysis.

Stage 2: Year 2012

All articles reported in this year fit in the grant and research topics and have in common qualitative analysis of some important classes of boundary value problems, thereby fulfilling all the objectives set for this stage. In the results section the publications are completely illustrated which underlie the description stage.

[1] G. Bonanno, G. Molica Bisci, V. Radulescu, Variational analysis for a nonlinear elliptic problem on the Sierpinski gasket, *ESAIM: Control, Optimization and Calculus of Variations*, DOI: 10.1051/cocv/2011199 (2011 Impact Factor: 0.758).

[2] M. Boureau, V. Radulescu, Anisotropic Neumann problems in Sobolev spaces with variable exponent, *Nonlinear Analysis* 75 (2012) 4471–4482 (2011 Impact Factor: 1.536).

[3] G. Bonanno, G. Molica Bisci, V. Radulescu, Nonlinear elliptic problems on Riemannian manifolds and applications to Emden–Fowler type equations, *Manuscripta Mathematica* DOI: 10.1007/s00229-012-0596-4 (2011 Impact Factor: 0.430).

[4] V. Radulescu, Multiple solutions for Lane-Emden equations with mixed nonlinearities, *Annals of the University of Bucharest (mathematical series)* 3 (LXI) (2012), 227-232.

Are considered: problem on domains with smooth boundary, and on fractals; Lane-Emden equations also nonlinear elliptic equations with variable exponent; problems containing classic operators like Laplace operator, and equations with non-standard operators such as nonhomogeneous differential operators; a classic functional framework, and problems in Orlicz spaces type. An important role is played by the applicability of all these results in areas as diverse as Riemannian geometry but also mathematical physics. Both problems are considered undisturbed and interesting phenomena generated by various types of disturbances. The techniques used in demonstrations are at the interface of complex methods of nonlinear functional analysis, variational calculation, asymptotic analysis and numerical analysis.

One of the many types of fields of fractals is Sierpinski carpet, which is a model for other irregular fields of this type.

Paper [1] is devoted to the analysis of a variational elliptic problem on Sierpinski's carpet. A first difficulty is related to the correct formulation of the problem. In this regard, recent studies related to the Laplace operator on fractals have been developed only in recent years by famous mathematicians. After the technical details of this type and rigorous formulation of the problem, we developed (through variational techniques) a general method to address many types of stationary nonlinear equations on fractals. Sufficient conditions are obtained for the existence of weak solutions

in optimal functional spaces. The methods developed in [1] are applicable to broad classes of problems on fractals.

Paper [2] is in connection with recent studies in robotics and non-Newtonian fluid theory related to the theory of differential operators with variable exponent. Another feature of paper [2] is related to the presence of Neumann boundary conditions corresponding to the energy exchange with the external environment, therefore appropriate to studied models. The techniques used in this paper are essentially variational and aim to establish results of existence of solutions in Sobolev spaces with variable exponent. The paper corresponds to anisotropic medium in relation to all directions, topic less studied in the literature, but very useful in applications.

The papers [3] and [4] are related and are considering qualitative study of one of the most important equations of mathematical physics, namely Lane-Emden-Fowler equation, which has many applications in various fields. The methods developed in these two papers allow both the study of disturbance phenomena and the study in a very general framework, corresponding to Riemann varieties. Disturbance phenomena are considering competition between different nonlinearities present in these equations and are based on a fine qualitative analysis of these processes. The geometric study is related to a very general framework approach, with many implications in various fields. The techniques combine variational analysis with topological methods and principles of comparison. An important role is also related to some recent results like the variational principle of Ricceri, which allows to obtain multiple solutions and their location depending on a parameter.

All the papers are qualify in the grant theme and continue the results of the previous year related to the study of non-standard phenomena in nonlinear analysis. Theoretical study is closely related to concrete models from nature and methods of approach are various. All the reported results are in close connection with the research interests of nonlinear analysis of major schools in France, Italy, Spain, USA, Slovenia, Hong Kong, etc. The problems studied in this year have generated some open issues that will be studied over the next year.

Stage 3: Year 2013

In 2013 the proposed objectives were followed, which are fully met and additional results were obtained, all are described in detail below and in the results section.

- 1) G. Bonanno, G. Molica Bisci, V. Radulescu, Weak solutions and energy estimates for a class of nonlinear elliptic Neumann problems, *Advanced Nonlinear Studies* 13(2013), 373-389 (2012 ISI Impact Factor: 0.538)
- 2) G. Bonanno, G. Molica Bisci, V. Radulescu, Qualitative analysis of gradient-type systems with oscillatory nonlinearities on the Sierpinski gasket, *Chinese Annals of Mathematics* 34B(2013), 381-398 (2012 ISI Impact Factor: 0.504)
- 3) V. Radulescu, Combined effects for a stationary problem with indefinite nonlinearities and lack of compactness, *Dynamic Systems and Applications* 22 (2013), 371-384
- 4) G. Molica Bisci, V. Radulescu, Multiple symmetric solutions for a Neumann problem with lack of compactness, *C. R. Acad. Sci. Paris Ser. I* 351 (2013), 37-42 (2012 ISI Impact Factor: 0.477)
- 5) N. Papageorgiou, V. Radulescu, Semilinear Neumann problems with indefinite and unbounded potential and crossing nonlinearity, in *Recent Trends in Nonlinear Partial Differential Equations. Dedicated to Patrizia Pucci on the Occasion of Her 60th Birthday*, *Contemporary Mathematics*, American Mathematical Society (J. Serrin, E. Mitidieri, V. Radulescu-Editors), vol. 595 (2013), 293-315
- 6) A. Iannizzotto, V. Radulescu, Positive homoclinic solutions for the discrete p -Laplacian with a coercive weight function, *Differential and Integral Equations*, in press (2012 ISI Impact Factor: 0.733)
- 7) G. Afrouzi, A. Hadjian, V. Radulescu, Variational analysis for Dirichlet impulsive differential equations with oscillatory nonlinearity, *Portugaliae Mathematica*, in press (2012 ISI Impact Factor: 0.422)
- 8) N. Papageorgiou, V. Radulescu, Qualitative phenomena for some classes of quasilinear elliptic equations with multiple resonance, *Applied Mathematics and Optimization*, in press (2012 ISI Impact Factor: 0.859)
- 9) G. Molica Bisci, V. Radulescu, Applications of local linking to nonlocal Neumann problems, *Communications in Contemporary Mathematics*, in press (2012 ISI Impact Factor: 0.753)
- 10) Ion Iancu, Nicolae Constantinescu, Intuitionistic Fuzzy System for Fingerprints Authentication, *Applied Soft Computing*, Vol. 13, Issue 4, pp. 2136-2142, 2013 (ISI Impact Factor: 2.612) (ISSN: 1568-4946, DOI: 10.1016/j.asoc.2012.11.001)

11) M.M. Boureau, C. Udrea, D.N. Udrea, Anisotropic problems with variable exponents and constant Dirichlet condition, 2013 (2013), Electron. J. Diff. Equ., no. 220, 1–13 (ISI Impact Factor 0.74) (ISSN: 1072-669, DOI: , Vol. 2013, No. 220).

12) M.M. Boureau, B. Noris, S. Terracini, Sub and supersolutions, invariant cones and multiplicity results for p-Laplace equations, Contemporary Mathematics – Recent Trends in Nonlinear Partial Differential Equations II: Stationary Problems. Dedicated to Patrizia Pucci on the Occasion of Her 60th Birthday, 595 (2013), 91–119.

Some of the results contained in this work were presented at prestigious scientific meetings such as:

(i) Meetings on Mathematics, University of Reggio Calabria, 24 ianuarie 2013, An overview of some research problems in applied nonlinear analysis (<http://www.unirc.it/architettura/articoli/10941/24-gennaio-meeting-on-mathematics>)

(ii) Université de Besancon, 28 februarie 2013, Problèmes de bifurcation revisites : cas d’opérateurs à exposant variable (http://lmb.univ-fcomte.fr/rubrique.php3?id_rubrique=82)

(iii) Université de Poitiers, 1 mai 2013, Bifurcation in anisotropic media (<http://rech-math.sp2mi.univ-poitiers.fr/fr/seminaires/sem2013-05.shtml>)

(iv) 5th Women in Mathematics Summer School, International Centre for Theoretical Physics, Trieste, 27 mai – 1 iunie 2013, Singular phenomena in life sciences described by nonlinear differential equations (<http://math.ucv.ro/~radulescu/articles/posterICTP.pdf>)

(v) Seminars on Nonlinear Analysis, University of Reggio Calabria, 22 octombrie 2013, Combined effects in nonlinear elliptic problems (<http://www.unirc.it/comunicazione/articoli/11944/22-ottobre-giornata-su-problemi-non-locali-di-tipo-laplaciano-frazionario>)

Also the director of this grant published as co-editor the following volumes in the prestigious series Contemporary Mathematics from the American Mathematical Society:

Recent Trends in Nonlinear Partial Differential Equations I: Evolution Problems, Contemporary Mathematics, vol. 594, American Mathematical Society, 307 pp., 2013 (<http://www.ams.org/bookstore?fn=20&arg1=whatsnew&ikey=CONM-594>)

Recent Trends in Nonlinear Partial Differential Equations II: Stationary Problems, Contemporary Mathematics, vol. 595, American Mathematical Society, 340 pp., 2013 (<http://www.ams.org/bookstore?fn=20&arg1=whatsnew&ikey=CONM-595>)

Selected Papers of James Serrin, Contemporary Mathematicians, Birkhäuser, Basel, 2 vol., 1600 pp., 2013 (<http://www.springer.com/birkhauser/history+of+science/book/978-3-0348-0685-5>)

All papers and activities above were conducted in accordance with commitments under the grant initial objectives and have dealt with qualitative analysis and numerical of several models at the interface between pure and applied nonlinear analysis. The results were published in prestigious journals and were communicated at conferences with high visibility in the community of mathematics or scientific seminars of western universities.

Paper [1] addresses in an original manner the study of weak solutions of nonlinear elliptic equation class with Neumann condition. By combining variational techniques energy estimates are obtained, leading to a better understanding of the nature of solutions.

Paper [2] aims at the study of nonlinear systems with gradient term and with nonlinearities of oscillatory type. The distinctive note is the analysis in a new frame, as well as on fractals of "Sierpinski's carpet" type. Numerous technical difficulties are overcome by combining several techniques at the confluence of nonlinear functional analysis, mathematical physics and variational calculation.

Paper [3] is focused on the analysis of the combined effect of several types of nonlinearities combined with lack of compactness in a nonlinear stationary problem. Many cases are addressed and results are determined by the existence or non-existence of the solution.

Paper [4] aims to study solutions with various symmetries for a Neumann problem class without compactness. Recent techniques are used of fixed point theory that allow qualitative description of the results obtained.

Article [5] is the beginning of a collaboration with a distinguished mathematician in Greece on issues related to the topic of this research grant. Strong topological methods are used to describe solutions, including for the analysis of the nodal solutions (which changes its sign).

Paper [6] is based on a problem of discrete equations and extends in a comprehensive framework non-trivial the results of Ambrosetti and Rabinowitz related to the equation of Lane-Emden. Variational analysis techniques are used.

Paper [7] deals with impulsive equations, this area is intensively studied in recent years. One of the main difficulties is related to the presence of nonlinear oscillators, in the vicinity of the origin and at infinity.

Paper [8] is a continuation of the article [5] and aims at studying the qualitative phenomena for some stationary problems at resonance. Topological methods are combined with the variational or monotony type in demonstrations.

Paper [9] treats a new topic, linking local theory to study Neumann problems. The method developed in this paper can be extended to multiple classes of nonlinear elliptic problems.

Paper [10] is an implementation of approximation methods for non-degenerate solutions of certain equations over elliptic curves.

Papers [11] and [12] treat special cases of invariants and multiple results (equations with variable exponent) for p-Laplace equations.

Stage 4: Year 2014

In this stage were treated, according to objectives, all points made, obtaining higher results than those set as follows in the description below and in accordance with the results section.

1. Technical and scientific description

Paper [1] describes a class of discrete problems with potential coercion. Variational techniques are used to determine sufficient conditions for the existence of positive homoclinic solutions. Critical point methods are used, including mountain pass theorem of Antonio Ambrosetti and Rabinowitz Paul plays a central role in the qualitative study of solutions. It is taken into account a large class of quasilinear problems respectively those described by p-Laplacian type operators. The techniques developed in this article can be applied to broader classes of boundary value problems, including those described by discrete operators with variable exponent.

Paper [2] deals with the qualitative analysis of solutions for a class of nonlinear problems with impulse of the fourth degree. The hypotheses imposed are very general and the techniques used are based on variational analysis solutions. The problems studied are inspired by real designs from mechanics and methods used in this paper can be extended to other classes of differential equations with one or more impulses.

Paper [3] deals with a subject of modern research or study a class of Navier-Stokes equations in spaces with variable exponent and through Clifford algebras. The case of stationary solutions is treated using appropriate techniques and Clifford algebras. The problem studied in this article is inspired by the relevant models from applied sciences and techniques developed in this paper are likely to be applied to other classes of stationary problems in Clifford algebras.

Paper [4] addresses a class of nonlinear elliptic problems described by general differential operators including anisotropic case. Are studied Dirichlet boundary condition problems and the results are original in terms of nonlinearities considered and oscillating around the origin and at infinity. In the first case solutions are obtained with low energy solutions, respectively norm tends to zero. In the case of nonlinear oscillatory terms infinitely large energy solutions are obtained, respectively the norm tending to infinity in a sense precisely described. Demonstrations are mainly based on the variational analysis combined with the analysis of the energy levels.

Paper [5] is focused on the study of resonance phenomena for nonlinear equations with Dirichlet boundary condition. Results are established by the existence or multiplicity of solutions and the demonstrations combine techniques of critical point and Morse theory (critical groups). These methods can be extended to other types of boundary value problems, respectively Neumann and Robin boundary conditions. Arguments longer use nonlinear principles of comparison like the Lyapunov-Schmidt reduction method.

Paper [6] deals with a very modern research topic namely nonlocal problems. Solutions of type mountain pass are searched and a variational method is developed for treating nonlocal type wider equations (fractional Laplacian and other operators). Based on this topic related to nonlocal problems, the authors of this article are currently under contract with Cambridge University Press to write the first book of nonlocal problems dealt with variational methods. Final manuscript will be drafted until June 2015.

Nonlinear Schrödinger equation is a EDP topic which is extensively studied in the past two decades.

Article [7] treats the stationary Schrödinger equation in a particular frame respectively the one corresponding to the variable exponents. The study is in Sobolev spaces with variable exponents and qualitative analysis methods of solutions are variational. In this way properties of existence are obtained and multiplicity of solutions.

Paper [8] combines Morse's theory with variational methods of linking type for qualitative analysis of degenerate classes of stationary problems arising in the theory of electrorheological fluids (non-Newtonian). Sufficient conditions are obtained for both for the existence of solutions and for obtaining multiplicity properties. These results were published in one of journals with the highest impact factor in mathematics.

Paper [9] is based on qualitative analysis of solutions of nonlinear elliptic problems with mixed condition of Robin type at the boundary. Results of multiplicity of solutions are obtained but also gives precise information on the sign of these solutions. In particular, it argues the existence of nodal solutions (which changes its sign). The methods developed here were developed by the same authors for other problems in several papers under review.

2. Results (selection)

Articles

- A. Iannizzotto, Vicentiu Radulescu, Positive homoclinic solutions for the discrete p-Laplacian with a coercive potential, *Differential and Integral Equations* 27 (2014), 35-44. [2013 ISI Impact Factor: 0.542]
- 2) G. Afrouzi, A. Hadjian, Vicentiu Radulescu, Variational approach to fourth-order impulsive differential equations with two control parameters, *Results in Mathematics* 65 (2014), 371-384. [2013 ISI Impact Factor: 0.642]
- 3) Y. Fu, Vicentiu Radulescu, B. Zhang, The stationary Navier-Stokes equations in variable exponent spaces of Clifford-valued functions, *Advances in Applied Clifford Algebras* 24 (2014), 231-252. [2013 ISI Impact Factor: 0.53]
- 4) G. Molica Bisci, Vicentiu Radulescu, R. Servadei, Low and high energy solutions of nonlinear elliptic oscillatory problems, *C. R. Acad. Sci. Paris, Ser. I* 352 (2014), 117-122. [2013 ISI Impact Factor: 0.425]
- 5) N. Papageorgiou, Vicentiu Radulescu, Qualitative phenomena for some classes of quasilinear elliptic equations with multiple resonance, *Applied Mathematics and Optimization* 69 (2014), 393-430. [2013 ISI Impact Factor: 0.681]
- 6) G. Molica Bisci, Vicentiu Radulescu, Mountain pass solutions for nonlocal equations, *Ann. Acad. Sci. Fennicae* 39 (2014), 579-592. [2013 ISI Impact Factor: 0.648]
- 7) G. Afrouzi, M. Mirzapour, Vicentiu Radulescu, Existence and multiplicity results for anisotropic stationary Schrödinger equations, *Rendiconti Lincei - Matematica e Applicazioni* 25 (2014), 91-108. [2013 ISI Impact Factor: 0.684]
- 8) Vicentiu Radulescu, B. Zhang, Morse theory and local linking for a nonlinear degenerate problem arising in the theory of electrorheological fluids, *Nonlinear Analysis: Real World Applications* 17 (2014), 311-321. [2013 ISI Impact Factor: 2.338]
- 9) N. Papageorgiou, Vicentiu Radulescu, Multiple solutions with precise sign for nonlinear parametric Robin problems, *Journal of Differential Equations* 256 (2014), 2449-2479. [2013 ISI Impact Factor: 1.57]
- 10) N. Papageorgiou, Vicentiu Radulescu, Bifurcation near infinity for the Neumann problem with concave-convex nonlinearities, *C. R. Acad. Sci. Paris, Ser. I* 352 (2014), 811-816. [ISI Impact Factor: 0,425]

Books

P. Pucci, V. Radulescu, H. Weinberger, Editors. James Serrin. Selected Papers, 2 volumes, 1718 pages, Contemporary Mathematicians, Birkhäuser, Basel, 2014.

Conferences

- (i) Vicentiu D. Radulescu, Nonstandard bifurcation phenomena in PDEs with variable exponent, University of East Anglia, Norwich, Applied Maths Research Seminars, 20.01.2014.
- (ii) Vicentiu D. Radulescu, Nonlinear eigenvalue problems for nonhomogeneous differential operators, Isaac Newton Institute for Mathematical Sciences, Cambridge, 22.01.2014.
- (iii) Vicentiu D. Radulescu, Anisotropic phenomena described by nonhomogeneous differential operators, Cardiff University, School of Mathematics, 29.01.2014.
- (iv) Vicentiu D. Radulescu, Low- and high-energy solutions of nonlinear elliptic oscillatory problems, Department of Mathematics, Swansea University, 30.01.2014.
- (v) Vicentiu D. Radulescu, Perturbation effects for nonlinear elliptic oscillatory equations, University of Nottingham, School of Mathematical Sciences, Algebra & Analysis Seminar, 31.01.2014.
- (vi) Vicentiu D. Radulescu, Nonlinear problems described by nonhomogeneous differential equations, Ecole de Recherche CIMPA & Workshop EDP Non Lineaires: Etude Theorique et Numerique, Tanger, 5-17 Mai 2014.
- (vii) Vicentiu D. Radulescu, Variational analysis of some problems with variable exponent, Dipartimento di Matematica, Universita di Pisa, 14 Mai 2014.
- (viii) Vicentiu D. Radulescu, Analysis on the Sierpinski gasket, Mini-symposium Recent Trends in Nonlinear Analysis and its Applications, 8th European Conference on Elliptic and Parabolic Problems, Gaeta, 26-30 Mai 2014.
- (ix) Vicentiu D. Radulescu, Nonlinear elliptic problems on the Sierpinski gasket, Recent Trends in Nonlinear Partial Differential Equations and Applications Celebrating Enzo Mitidieri's 60th Birthday, University of Trieste, 28-30 Mai 2014.
- (x) Vicentiu D. Radulescu, A sharp characterization on fractal sets, International Workshop on Nonlinear Analysis and Applications to Economics Dedicated to Professor Dusan Repovs on his 60th birthday, Craiova, 25 Septembrie 2014.

(xi) Vicentiu D. Radulescu, Variational analysis on fractals, Recent Trends on Nonlinear Phenomena, Reggio Calabria, 5-7 Noiembrie 2014.

(xii) Vicentiu D. Radulescu, Morse theory and multiple solutions of Robin problems, Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia, 1 Decembrie 2014.

Volumes editor/co-editor

Guest Editor of the Special Issue Degenerate and Singular Phenomena in Nonlinear Analysis, Nonlinear Analysis: Theory, Methods and Applications, 2014.

Research Reports

1) Vicentiu Radulescu, A. Bahrouni and H. Ounaies, Infinitely many solutions for a class of sublinear Schrödinger equations with indefinite potentials, Isaac Newton Institute, University of Cambridge, NI14008-FRB/2014.

2) Vicentiu Radulescu and B. Alleche, Equilibrium problems techniques in the qualitative analysis of quasi-hemivariational inequalities, Isaac Newton Institute, University of Cambridge, NI14009-FRB/2014.

3) Vicentiu Radulescu, B. Alleche and M. Sebaoui, The Tikhonov regularization for equilibrium problems and applications to quasi-hemivariational inequalities, Isaac Newton Institute, University of Cambridge, NI14044-FRB/2014.

4) Vicentiu Radulescu and M. Malin, Competition phenomena for difference equations with oscillatory nonlinearities, Isaac Newton Institute, University of Cambridge, NI14050-FRB/2014.

Stage 5: Year 2015

In this stage were treated, according to objectives, the qualitative and quantitative newest problems on nonlinear elliptic analysis, obtaining higher results than those set as follows in the description below and in accordance with the results section, with applications in information theory.

Scientific Description

1. Paper [1] studies the qualitative analysis of solutions for a class of nonlinear elliptic problems with condition. Neumann at the boundary. The main characteristic is the presence unbounded potential which can change its sign. Using the variational and topological arguments the existence of weak solutions is shown in an adequate functional space.

2. Article [2] continues the study nonlinear elliptic problems of Neumann type. The main instrument is tied to the theory on the critical point, respectively methods of type *local linking*, which were developed in nonlinear analysis in the same time with Paul Rabinowitz pioneering papers. The methods developed in this paper can be extended to other classes of equations with nonlinear partial derivatives like the one described by the inhomogeneous differential operators.
3. Paper [3] is tied to fractal analysis, which are domains with smooth boundaries and to which, in the last two decades, a lot of development was made in the same time with the pioneering papers of R. Strichartz. The paper presents a characterization result in the case of problems with nonlinearity with sublinear growth. A strong argument in the analysis developed in this article is the recent variational principle of Ricceri.
4. In [4] it is studied the combined effect of more nonlinearities for problems of type Robin or Neumann. The paper has at his basis the analysis of some bifurcation phenomena and the results obtained describe in an exhaustive manner the phenomena which takes place into a very general theoretic context.
5. Work [5] is focused on the study of singular phenomenon from nonlinear elliptic equation theory. The demonstrations are based on variational techniques and analytical methods, combined with inequalities of type Caffarelli-Kohn-Nirenberg, which extend the classic inequalities of Sobolev.
6. Paper [6] continues the study of nonlinear Schrodinger equations, mentioning that it is taken into consideration the presence of some potentials which change their sign. The article sets sufficient conditions for solution existence, and there are established qualitative properties of the solutions.
7. Paper [7] is tied to Sturm-Liouville nonlinear theory and the main characteristic is the fact that the nonlinear term contains the derivative contribution. Using variational methods there are established sufficient conditions for the existence of solutions.
8. Article [8] is in connection with phenomenon of type Ambroseti-Brezis-Cerami for nonlinear problems of elliptic type. The main contribution of this paper is the presence of many nonlinearities with variational exponent, which corresponds to anisotropic phenomenon which take place in non-Newtonian fluid theory.
9. Paper [9] is a survey of some recent results concerning equations with variable exponent, also a contribution to the study of this phenomenon with great impact in applied science. There are studied classical problems of partial derivatives equations and problems that contain the discrete Laplace operator with variable exponent.
10. Work [10] is focused on a recent problem, respectively which contains both operators of type Laplace. The studies of the competition between these two differential operators highlights the appearance of new phenomenon. The qualitative analysis developed in the paper combines topological and variational methods.
11. Paper [11] continues the study of convex-concave elliptic problems with undefined potential. Starting from the rigorous analysis of the energetic levels of the attached functional, there are established more results of multiplicity.
12. Article [12] is tied to the existence of solutions with minimum energy (*ground state*) for a wide class of nonlinear elliptic equations. Variational, analytical and topological techniques

are used. The paper was published in a journal of nonlinear analysis which has the greatest visibility.

3. Scientific Articles. Publications

1. N. Papageorgiou, Vicentiu Radulescu, Resonant Neumann problems with indefinite and unbounded potential, *Applied Mathematics Letters* 40 (2015), 49-52 (2014 ISI Impact Factor: 1.337)
2. G. Molica Bisci, Vicentiu Radulescu, Applications of local linking to nonlocal Neumann problems, *Communications in Contemporary Mathematics* 1/17 (2015), 1450001 (17 pages) (2014 ISI Impact Factor: 0.835)
3. G. Molica Bisci, Vicentiu Radulescu, A characterization for elliptic problems on fractal sets, *Proceedings Amer. Math. Soc.* 143 (2015), 2959-2968 (2014 ISI Impact Factor: 0.681)
4. N. Papageorgiou, Vicentiu Radulescu, Bifurcation of positive solutions for nonlinear nonhomogeneous Robin and Neumann problems with competing nonlinearities, *Discrete and Continuous Dynamical Systems* 35 (2015), 5003-5036 (2014 ISI Impact Factor: 0.826)
5. G. Afrouzi, Vicentiu Radulescu, S. Shakeri, Positive solutions of singular elliptic systems with multiple parameters and Caffarelli-Kohn-Nirenberg exponents, *Periodica Mathematica Hungarica* 70 (2015), 145-152 (2014 ISI Impact Factor: 0.479)
6. A. Bahrouni, H. Ounaies, Vicentiu Radulescu, Infinitely many solutions for a class of sublinear Schrödinger equations with indefinite potentials, *Proceedings of the Royal Society of Edinburgh: Section A (Mathematics)* 145A (2015), 445-465 (2014 ISI Impact Factor: 1.009)
7. G. Afrouzi, A. Hadjian, Vicentiu Radulescu, A variational approach of Sturm-Liouville problems with the nonlinearity depending on the derivative, *Boundary Value Problems* (2015), 2015:81, 17 pp. (2014 ISI Impact Factor: 1.014)
8. Vicentiu Radulescu, I. Stancut, Combined concave-convex effects in anisotropic elliptic equations with variable exponent, *Nonlinear Differential Equations and Applications (NoDEA)* 22 (2015), 391-410 (2014 ISI Impact Factor: 0.897)
9. Vicentiu Radulescu, Nonlinear elliptic equations with variable exponent: old and new, *Nonlinear Analysis: Theory, Methods and Applications* 121 (2015), 336-369 (2014 ISI Impact Factor: 1.327)
10. N. Papageorgiou, Vicentiu Radulescu, Resonant $(p,2)$ -equations with asymmetric reaction, *Analysis and Applications* 13 (2015), 481-506 (2014 ISI Impact Factor: 0.796)

11. N. Papageorgiou, Vicentiu Radulescu, Combined effects of concave-convex nonlinearities and indefinite potential in some elliptic problems, *Asymptotic Analysis* 93 (2015), 259-279 (2014 ISI Impact Factor: 0.528)
12. G. Molica Bisci, Vicentiu Radulescu, Ground state solutions of scalar field fractional Schrödinger equations, *Calculus of Variations and Partial Differential Equations* 54 (2015), 2985-3008 (2014 ISI Impact Factor: 1.518)