
Recent Advances in Nonlinear Analysis,
on the occasion of 60th birthday of Vicentiu Radulescu

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BOOK OF ABSTRACTS / TALKS

INVITED TALKS

Theorems on the existence of solutions of the operator equations in WC - Banach algebras and their applications

JOZEF BANAS

Rzeszow University of Technology (Poland)

We present a few theorems on the existence of solutions of the operator equation having the form

$$x = AxBx + Cx. \quad (*)$$

In our considerations we assume that X is the so-called WC-Banach algebra i.e., X is a Banach algebra such that the product (in the sense of the product in the algebra X) of arbitrary two weakly compact sets is weakly compact in X .

We assume that $A, C : X \rightarrow X$ are operators satisfying the condition (H_2) (which means that the image of any weakly convergent sequence under operators A, C contains a weakly convergent subsequence) and these operators are D-Lipschitzian i.e., they satisfy a generalized Lipschitz condition. Moreover, the operator A is regular on X (this means that Ax is invertible for each $x \in X$). We also assume that the operator B is defined and continuous on a nonempty, bounded, closed and convex subset S with values in X and the set BS is relatively weakly compact. Additionally, we assume that there is satisfied the following Krasnosel'skii's type condition

$$x = AxBy + Cx \implies x \in S$$

for every $y \in S$.

Under above assumptions (and some others having the technical character) it is shown that equation $(*)$ has a solution in the set X .

In order to illustrate the above result we discuss theorem on the existence of solutions of the infinite system of nonlinear integral equations of the form

$$x_n(t) = c_n(x_n(t)) + a_n(x_n(t)) \int_0^1 b(t, s) f_n(s, x_n(s), x_{n+1}(s), \dots) ds,$$

where $t \in [0, 1]$ and $n = 1, 2, \dots$

On the effect of the equivalence $\mathbf{p} \cong \mathbf{v}^2$ on the classical L-P-S conditions for regularity of the Navier-Stokes equations

HUGO BEIRÃO DA VEIGA

Università Pisa (Italy)

The subject of these talk is the well known sufficient condition for regularity of solutions to the evolution Navier-Stokes equations sometimes called Ladyzhenskaya-Prodi-Serrin's condition (LPS condition), namely $v \in L^r(0, T; L^q(\Omega))$, where $2/r + n/q = 1$ and $q > n$. On the

other hand, an equivalence relation $\mathbf{p} \cong |\mathbf{v}|^2$ is suggested by the Navier-Stokes equations themselves. In three papers published nearly twenty years ago we have considered some effects of the above “equivalence” on the LPS assumption. In a recent paper we obtained new results in this direction. Many open problems remain.

Surfaces of prescribed mean curvature in central projection - a variational approach

JOSEF BEMELMANS

RWTH Aachen University (Germany)

Minimal surfaces in central projection were first studied by T. Radó in 1932, and in the extended field of surfaces of prescribed mean curvature also graphs in central projection were investigated. In this talk such graphs are studied as minimizers to a variational integral that is defined for functions of bounded variation. This is joint work with Jens Habermann, University of Erlangen-Nürnberg.

Classical solutions of the divergence and curl equation with Dirichlet conditions

LUIGI C. BERSELLI

Università di Pisa (Italy)

We consider the boundary value problem associated to the divergence and curl operator with vanishing Dirichlet boundary conditions and we prove the existence of classical solutions for Dini continuous data.

On the Leray-Lions type operators and some elliptic problems

MARIA-MAGDALENA BOUREANU

University of Craiova (Romania)

Introduced by J. Leray and J.-L. Lions, the differential operators that bear the names of these famous mathematicians have proved to be not only useful, but also quite versatile. Thus our aim is to present and discuss different classes of nonlinear elliptic problems involving Leray-Lions type operators. We are concerned with the weak solvability of such problems and the discussion is conducted in the framework of the spaces with variable exponent.

New Critical Phenomena in Nonlinear Nonlocal Schrödinger equations

DANIELE CASSANI

Università dell'Insubria, Varese (Italy)

We are concerned with a class of nonlocal Schrödinger equations which show up in many different applied contexts. In particular we consider the case in which the nonlinear interaction has critical features. In the classical Sobolev sense, when we are in presence of the Hardy-Littlewood-Sobolev upper-critical exponent, as well as new critical phenomena occur, the so-called 'bubbling at infinity', when the nonlinearity exhibits lower-critical growth. Assuming mild conditions on the nonlinearity and by using variational methods, we establish existence, non-existence and qualitative properties of finite energy solutions. Some partial results in the limiting case of dimension two will be also presented.

(Joint works in collaboration with: J. Zhang; J. Van Schaftingen and J. Zhang; C. Tarsi and M. Yang).

On nonlocal fractional Laplacian problems with oscillating potential

LUIGI D'ONOFRIO

Università Parthenope, Napoli (Italy)

In this talk I will present a joint work with Vincenzo Ambrosio and Giovanni Molica Bisci in which we study the following fractional nonlocal p -Laplacian problem

$$\begin{cases} (-\Delta)_p^s u = \lambda\beta(x)u^q + f(u) & \text{in } \Omega \\ u \geq 0, u \not\equiv 0 & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $\Omega \subset \mathbf{R}^N$ is a bounded domain with smooth boundary of \mathbb{R}^{N^N} , $s \in (0, 1)$, $p \in (1, \infty)$, $N > sp$, λ is a real parameter, $\beta \in L^\infty(\Omega)$ is allowed to be indefinite in sign, $q > 0$ and $f : [0, +\infty) \rightarrow \mathbb{R}$ is a continuous function oscillating near the origin or at infinity. By using variational and topological methods, we obtain the existence of infinitely many solutions for the problem under consideration. The main results obtained here represent some new interesting phenomena that are new in the nonlocal setting.

Existence of solutions for quasilinear Dirichlet problems with gradient terms

ROBERTA FILIPPUCCI

Università di Perugia (Italy)

In this talk we discuss an existence theorem for positive solutions of a nonlinear Dirichlet problem involving the p -Laplacian operator on a smooth bounded domain when a nonlinearity depending on the gradient is considered. Our main theorem extends a previous result by Ruiz in J. Differential Equations (2004), in which a slight modification of the celebrated blowup technique due to Gidas and Spruck in Comm. Pure Appl. Math (1981), is introduced.

Isolated singularities for semilinear elliptic inequalities

MARIUS GHERGU

University College Dublin (Ireland)

We report on some recent results relates to semilinear elliptic inequalities in a punctured domain. We discuss the existence of pointwise bounds and derive optimal ranges on exponents for such bounds to exist.

This talk is based an joint works with Steven D. Taliaferro (Texas A&M University).

Fractional and singular elliptic equations

JACQUES GIACOMONI

Université de Pau (France)

In this talk, we study the positive solutions to the following singular and non local elliptic problem posed in a bounded and smooth domain $\Omega \subset \mathbb{R}^N$, $N > 2s$:

$$(P_\lambda) \begin{cases} (-\Delta)^s u = \lambda(K(x)u^{-\delta} + f(u)) \text{ in } \Omega \\ u > 0 \text{ in } \Omega \\ u \equiv 0 \text{ in } \mathbb{R}^N \setminus \Omega. \end{cases}$$

Here $0 < s < 1$, $\delta > 0$, $\lambda > 0$ and $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a positive C^2 function. $K : \Omega \rightarrow \mathbb{R}^+$ is a Hölder continuous function in Ω which behave as $\text{dist}(x, \partial\Omega)^{-\beta}$ near the boundary with $0 \leq \beta < 2s$.

First, for any $\delta > 0$ and for $\lambda >$ small enough, we prove the existence of solutions to (P_λ) .

Next, for a suitable range of values of δ , we show the existence of an unbounded connected branch of solutions to (P_λ) emanating from the trivial solution at $\lambda = 0$. For a certain class of nonlinearities f , we derive a global multiplicity result that extends results proved in [1]. To establish the results, we prove new properties which are of independent interest and deal with the behavior and Hölder regularity of solutions to (P_λ) .

References

[1] Adimurthi, J. Giacomoni and S. Santra: Positive solutions to a fractional equation with a singular nonlinearity, accepted in Journal of Differential Equations.

Regularity of extremal solutions of a Liouville system

OLIVIER GOUBET

Université de Picardie, Amiens (France)

Let $\Omega \subset \mathbb{R}^n$ be a bounded smooth open set. We prove that the extremal solution of the system

$$-\Delta u = \mu e^{\theta u + (1-\theta)v}, \quad -\Delta v = \lambda e^{\theta v + (1-\theta)u} \quad \text{in } \Omega,$$

with $u = v = 0$ on $\partial\Omega$, θ in $[0, 1]$ and $\mu, \lambda \geq 0$ are smooth if $n \leq 9$.

Elliptic and parabolic equations under general and p, q growth conditions

PAOLO MARCELLINI

Università di Firenze (Italy)

We give some recent *existence and interior regularity results* - partly obtained in collaboration with *Giovanni Cupini, Michela Eleuteri and Elvira Mascolo* - for elliptic partial differential equations in divergence form, or elliptic systems of m partial differential equations in divergence form of the type

$$\sum_{i=1}^n \frac{\partial}{\partial x_i} a_{\alpha}^i(x, u(x), Du(x)) = b_{\alpha}(x, u(x), Du(x)), \quad \alpha = 1, 2, \dots, m,$$

for maps $u : \Omega \subset \mathbb{R}^n \rightarrow \mathbb{R}^m$. Here the vector field $(a_{\alpha}^i(x, s, \xi))$ assumes values in the set of $m \times n$ matrices and it satisfies some *general growth conditions* with respect to the gradient variable $\xi \in \mathbb{R}^{m \times n}$, sometime *p, q growth conditions*.

As a part of a joint research-project with *Verena Bögelein and Frank Duzaar*, we consider the evolution problem associated with a convex integrand $f : \mathbb{R}^{m \times n} \rightarrow [0, \infty)$ satisfying - for instance - some *p, q -growth assumption*. To establish the existence of solutions we introduce the concept of *variational solutions*. In contrast to weak solutions, i.e. mappings $u : \Omega_T \subset \mathbb{R}^{n+1} \rightarrow \mathbb{R}^m$ which solve

$$\partial_t u - \operatorname{div} Df(Du) = 0$$

weakly in Ω_T , variational solutions in general exist under a much weaker assumption on the gap $q - p$.

In particular, if $2 \leq p \leq q < p + \min\{1, \frac{4}{n}\}$, we obtain the existence of variational solutions and we also show that they are actually - in this case - weak solutions. This means that any solution u automatically admits the necessary higher integrability of the spatial derivative Du to satisfy the parabolic system in the weak sense, i.e. we prove that

$$u \in L_{\text{loc}}^q \left(0, T; W_{\text{loc}}^{1,q}(\Omega, \mathbb{R}^m) \right).$$

Quasilinear elliptic system in divergence form associated to general nonlinearities

ENZO MITIDIERI

Università di Trieste (Italy)

We present recent results on a-priori estimates of positive solutions of a class of quasilinear elliptic systems associated to general nonlinearities and some consequences.

A flower-shape result in Heisenberg groups

GIOVANNI MOLICA BISCI

Università di Reggio Calabria (Italy)

In this talk some existence and multiplicity results for subelliptic equations on Carnot groups will be presented. The main tools are some group-theoretical arguments and the Haar measure theory on stratified Lie algebras.

The talk is mainly based on some recent joint works with P. Pucci as a part of a joint research-project started in 2017.

On fractional plasma problems

DIMITRI MUGNAI

Università della Tuscia, Viterbo (Italy)

We show existence and multiplicity results for nonlocal nonlinear elliptic problems modelling the flux of a plasma surrounded by a vacuum layer in a Tokamak machine.

Dirichlet $(p, 2)$ -equations

NIKOLAOS S. PAPAGEORGIOU

National Technical University of Athens (Greece)

We consider nonlinear Dirichlet problems driven by the sum of a p -Laplacian and a Laplacian (a $(p, 2)$ equation). Using variational and Morse theoretic arguments we prove multiplicity results for such equations under various conditions on the reaction term.

Hardy systems with critical nonlinearities

PATRIZIA PUCCI

Università di Perugia (Italy)

The talk deals with the existence of nontrivial entire solutions for critical Hardy quasilinear systems driven by general (p, q) elliptic operators of Marcellini types. Existence is derived as an application of the mountain pass geometry and the concentration-compactness principle of Lions. The constructed solution has both components nontrivial, that is it solves the actual system, which does not reduce into an equation.

The importance of studying problems involving (p, q) operators, or operators with non-standard growth conditions, begins with the pioneering papers of Marcellini and Zhikov. Since then the subject has been attracting increasing attention on existence, regularity and qualitative properties of solutions of different problems.

The assumptions we use are milder and, in any case, much different from the usual requests granted in related problems, even if the lack of compactness, due to the presence of the Hardy terms and the critical nonlinearities, makes the search of entire solutions fairly delicate. Finally, the results improve or complement previous theorems for the quasilinear (p, q) scalar as well as vectorial problems.

Vicentiu Radulescu: Some key facts about his life and professional career

DUŠAN D. REPOVŠ

University of Ljubljana (Slovenia)

In this talk we shall look at the life and work of the jubilant, Professor Vicentiu Radulescu. We shall begin by describing his formative years, growing up in Caracal, Romania, underlining the enormous lifetime importance of the most loving care and undivided support of his dear parents, Professor Dumitru Radulescu (1914-1982) and Primary School Teacher Ana Radulescu (1923-2011). We shall continue by presenting his formation period, first studying in Romania and then continuing in Paris, where he defended Habilitation a Diriger des Recherches under the guidance of the celebrated Professor Haim Brezis. In the second part of the talk we shall look at his professional career, both as a researcher and winner of many domestic and scientific awards, as well as a university professor and administrator. We shall also mention his family which has always provided him with the strongest love and support.

Optimal Control in Computational Hemodynamics

ADÉLIA SEQUEIRA

University of Lisbon (Portugal)

Blood flow simulations can be improved by integrating known data into the numerical simulations. Data Assimilation techniques based on a variational approach play an important role in this issue. We propose a non-linear optimal control problem to reconstruct the blood flow profile from partial observations of known data in different geometries. To simplify, blood flow is assumed to behave as a Navier-Stokes fluid. Using a Discretize then Optimize (DO) approach, we solve a non-linear optimal control problem and present numerical results that indicate its robustness with respect to different idealized geometries and measured data. Blood flow in real vessels will also be considered, including the discussion of a particular clinical case.

Joint work with J. Tiago and T. Guerra.

Nonlocal critical problems with lack of compactness

RAFFAELLA SERVADEI

Università di Urbino (Italy)

Motivated by the interest shown in the literature for nonlocal operators of elliptic type, in recent years we considered various problems in this setting. In this talk we will present recent existence and multiplicity results for nonlocal fractional equations in presence of critical nonlinearity. These results, which extend the validity of some theorems known in the classical case of the Laplacian to the nonlocal framework, were obtained via variational and topological methods.

A critical fractional Kirchhoff-type problem involving discontinuous nonlinearity

BINLIN ZHANG

Heilongjiang Institute of Technology, Harbin (P.R. China)

In this talk, we first give a fractional version of the principle of concentration compactness in the setting of fractional p -Laplacian. Then we are concerned with the existence and multiplicity of solutions for the fractional p -Kirchhoff type problems involving critical exponent and discontinuous nonlinearity. It is worthy pointing out that our problems possess significant difficulties due to the presence of the possibly degenerate Kirchhoff function and of the nonlocal feature of the fractional Laplacian. Under the circumstance, some existence results are obtained when the nonlinearity is discontinuous.

MINI-SYMPOSIUM

Weighted shifts acting on directed semi-trees

WITOLD MAJDAK

AGH University of Science and Technology, Krakow (Poland)

It will be shown how the notion of a weighted shift acting on a directed tree can be extended to the case of a more general graph which we call a directed semi-tree. Some basic properties of such operators will be investigated. It will be shown that a generalized creation operator on the Segal-Bargman space can be represented as a weighted shift on a directed semi-tree of a particular form.

A measure of noncompactness in the space of functions with tempered increments on the half-axis

RAFAL NALEPA

Rzeszow University of Technology (Poland)

We will consider the Banach space $C_\omega(\mathbb{R}_+)$ consisting of functions defined on the interval $\mathbb{R}_+ = [0, \infty)$ and having increments tempered by a given modulus of continuity i.e., such that $|x(t) - x(s)| \leq k_x \omega(|t - s|)$ for $t, s \in \mathbb{R}_+$, where $k_x > 0$ is a constant depending on x . Moreover, $\omega : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ (called the modulus of continuity) is a function which is nondecreasing on \mathbb{R}_+ , $\omega(0) = 0$, $\omega(\varepsilon) > 0$ for $\varepsilon > 0$ and ω is continuous at the point $\varepsilon = 0$. The space $C_\omega(\mathbb{R}_+)$ is equipped with the norm $\|\cdot\|_\omega$ defined by the formula

$$\|x\|_\omega = |x(0)| + \sup \left\{ \frac{|x(t) - x(s)|}{\omega(|t - s|)} : t, s \in \mathbb{R}_+, t \neq s \right\}.$$

We formulate a condition sufficient for relative compactness of bounded subset of the space $C_\omega(\mathbb{R}_+)$. Based on that condition we construct a measure of noncompactness in the space in question. A few properties of that measure will be presented and its applications will be indicated.

Geometric Aspects of P-capacitary potentials

ANDREA PINAMONTI

Università di Trento (Italy)

The aim of this talk is to provide monotonicity formulas for solutions to the p-Laplace equation defined in the exterior of a convex domain. A number of analytic and geometric consequences are derived as well as new characterizations of rotationally symmetric solutions and domains. The talk is based on a joint work L. Mazziari and M. Fogagnolo from the University of Trento.

Well-posed minimization problems with respect to measures of noncompactness

TOMASZ ZAJAC

Rzeszow University of Technology (Poland)

The main goal of the presentation is to discuss the well-posed minimization problems for lower bounded and lower semicontinuous functionals defined on a closed subset of a metric space. We call attention to the well-posedness of minimization problems for functionals defined on a closed subset of a Banach space from the point of view of the theory of measures of noncompactness. The idea of the well-posedness of minimization problems comes back to A. N. Tikhonov (1966) and was subsequently developed by E. S. Levitin and B. T. Polyak (1966) and next by M. Furi and A. Vignoli (1970).

Sign-changing Solutions of Nonlinear Schrödinger-Poisson Systems in \mathbb{R}^3

JIANJUN ZHANG

Chongqing Jiaotong University (P.R. China)

We discuss the sign-changing solutions of the Schrödinger-Poisson system

$$\begin{cases} -\Delta u + V(x)u + \phi u = f(u) & \text{in } \mathbb{R}^3, \\ -\Delta \phi = u^2 & \text{in } \mathbb{R}^3. \end{cases}$$

By using the method of invariant sets of descending flow, we prove that this system has multiple sign-changing solutions. In particular, the nonlinear term includes the power-type nonlinearity $f(u) = |u|^{p-2}u$ for the well-studied case $p \in (4, 6)$ and the less studied case $p \in (3, 4)$, and for the latter case few existence results are available in the literature.

This is joint work with Prof. Zhaoli Liu and Prof. Z.-Q. Wang.
