Proiect:

Contract PCE 23/2017, Cod Depunere: *PN-III-P4-ID-PCE-2016-0130*

Contractor: Universitatea din Craiova

Director Proiect: Prof. univ. dr. Vicentiu Radulescu

I. Rezumat Raport Stiintific si Rezultate Obtinute

In cadrul proiectului de cercetare PCE 23/2017 au fost efectuate studii in domeniul matematici fundamentale cat si aplicati concrete, combinand analiza calitativa a problemelor propuse descrise de operatori diferentiali neomogeni cu analiza numerica a solutiilor unor clase largi de sisteme neliniare. Tematica s-a aflat la interfata dintre analiza neliniara pura si cea aplicata, fizica matematica si analiza numerica. Instrumentele matematice au combinat tehnici rafinate de topologie, calcul variational, ecuatii neliniare diferentiale si cu derivate partiale, geometrie diferentiala, analiza functionala si armonica si analiza numerica. Aceasta intrepatrundere arata complexitatea problematicii abordate si reprezinta o sursa aparent inepuizabila de probleme deschise care sunt in stransa legatura cu modele concrete din stiintele aplicate.

In cadrul studiilor efectuate au fost indeplinite toate clauzele asumate, au fost obtinute rezultate suplimentare si s-au creat premizele viitoarelor studii precum si atragerea de noi cercetatori in domeniul matematicii fundamentale cat si aplicate.

Rezultatele obtinute au fost publicate in Jurnale de prestigiu international cotate ISI, cu Factor de Impact. In acest sens putem puncta:

1. Publicarea unui numar de **44 articole indexate ISI**, cu factor de impact, in jurnale de prestigiu international. O selectie a acestor publicatii este evidentiata in listele de articole publicate in fiecare an, descrisa in capitolele urmatoare
2. Obtinerea unui **Factor de Impact** cumulat de **49,975 puncte**
3. Publicarea unui numar de **2 carti** in **edituri internationale** de prestigiu, conform listelor anuale descrise mai jos
4. Obtinerea a **7 pozitii de profesor invitat**, ca recunoastere a semnificatiei si importantei rezultatelor obtinute
5. Un numar de **14 conferinte invitate**

Dincolo de orice cuantificare a rezultatelor, calitatea rezultatelor stiintifice obtinute rezida in larga lor aplicabilitate, precum si in invitatiile de colaborare cu membri ai unor renumite scoli de matematica fundamentala si aplicata. Aceste aspecte au creat o larga deschidere internationala a echipei de cercetare si a scoli romanesti de matematica.

II. Etapa 1 – 2017

1. Rezumatul etapei

In cadrul acestei etape au fost realizate studii ale proprietatilor calitative ale spectrului de operatori poliarmonici cu exponent variabil, a fost efectuat studiul fenomenelor spectrale ale operatorilor Laplace cu exponent variabil:

1. Studiul existentei spectrului continuu
2. Studiul posibilelor fenomene de concentrare in vecinatatea originii sau la infinit
3. Extinderea studiilor la alte clase de operatori diferentiali cu exponent variabil

Conform rezultatelor obtinute, au fost realizate toate obiectivele propuse, evidentiindu-se crearea unor premize pentru depasirea rezultatelor initial propuse, prin evidentierea unui aparat matematic imbunatatit fata de asumarile initiale si optimizari ale sistemelor de implementare datorate DataRoom-ului existent in Laboratorul de Cercetare MANA (http://stiinte.ucv.ro/mana/) si atragerea in cadrul echipei a noi colaboratori din tara si strainatate.

2. Descrierea stiintifica si tehnica

Aceste studii au avut ca rezultat publicarea de articole stiintifice in Jurnale de prestigiu International, participari la Conferinte Internationale si pozitii de Profesor Invitat la Institutii de recunoastere mondiala in domeniul studiat, dupa cum urmeaza in descrierea de mai jos:

3. Lista cu articolele publicate (selectie)

1. N. Papageorgiou, **Vicentiu Radulescu,** Superlinear, noncoercive asymmetric Robin problems with indefinite, unbounded potential, *Z. Anal. Anwend.* 36 (2017), 253-281
   DOI: 10.4171/ZAA/1588
   Impact Factor: 0.643
   Descriere. Se considera o clasa de probleme semiliniare cu un potential indefinit si nemarginit si cu o conditie de tip Robin pe frontiera. Termenul de reactive are un comportament asimetric, respectiv este superliniar in directia pozitiva (dar fara a satisface conditia Ambrosetti-Rabinowitz) si este subliniar si necoerciv in directia negativa. Folosind metode variationale, tehnici de perturbare si teoria Morse, se demonstreaza existenta a cel putin trei solutii netriviale, doua avand semn constant si a treia solutie fiind nodala.

2. N. Papageorgiou, **Vicentiu Radulescu,** Periodic solutions for time-dependent subdifferential evolution inclusions, *Evolution Equations and Control Theory* 6 (2017), 277-297
   doi:10.3934/eect.2017015
   Impact Factor: 0.826
   Descriere. In acest articol se analizeaza existenta solutiilor periodice pentru o clasa de incluziuni neliniare de evolutie. Se obtin rezultate de existenta atat pentru problema convexa cat si pentru cea neconvexa. In particular, se demonstreaza existenta traiectoriilor extremale, respectiv solutii ce trec prin punctele extremale ale perturbarii multivoce. In lucrare se demonstreaza si faptul ca fiecare solutie a problemei convexe poate fi aproximata in norma sup cu solutii extremale. Mai multe exemple ilustreaza principalele rezultate din aceasta lucrare.

3. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Robin problems with indefinite linear part and competition phenomena, *Communications on Pure and Applied Analysis* 16 (2017), 1293-1314
   doi:10.3934/cpaa.2017063
   Impact Factor: 0.801
   Descriere. Se considera o clasa de probleme semiliniare parametrice de tip Robin cu potential nedefinit. Termenul de reactie include efecte de competitive intre mai multe tipuri de neliniaritati, respectiv este suma dintre un termen parametric subliniar (concav) si un termen superliniar (convex). Termenul superliniar nu este descris de o conditie de crestere de tip Ambrosetti-Rabinowitz. Se demonstreaza o teorema generala de bifurcatie. In plus, se arata existenta unei solutii minimale precum si properietati de monotonie si continuitate ale acestei solutii in raport cu parametrul de bifurcatie.

4. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Multiple solutions for resonant problems of the Robin *p*-Laplacian plus an indefinite potential, *Calculus of Variations and PDEs* (2017) 56:63
   DOI 10.1007/s00526-017-1164-2
   Impact Factor: 1.532
   Descriere. Se studiaza o clasa de probleme quasiliniare cu termen de reactie de tip Caratheodory si cu potential nedefinit. Neliniaritatea este asimptotic rezonanta la infinit in raport cu o valoare proprie neprincipala obtinuta prin teoria Ljusternik-Schnirelmann. Folosind metode variationale combinate cu teorie Morse si tehnici de perturbare, se arata ca problema are cel putin trei solutii netede netriviale, dintre care doua au semn constant.

5. N. Papageorgiou, **Vicentiu Radulescu,** An infinity of nodal solutions for superlinear Robin problems with an indefinite and unbounded potential, *Bull. Sci. Math.* 141 (2017), 251-266 http://dx.doi.org/10.1016/j.bulsci.2017.03.001
   Impact Factor: 0.75
   Descriere. In aceasta lucrare se considera o clasa de probleme semiliniare de tip Robin cu potential nemarginit. Folosindu-se o versiune a teoremei mountain pass a lui Ambrosetti si Rabinowitz, se demonstreaza existenta unei familii infinite de solutii, ale caror energii diverg catre infinit (solutii cu energie inalta).

6. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Robin problems with a general potential and a superlinear reaction, *Journal of Differential Equations* 263 (2017), 3244-3290

   http://dx.doi.org/10.1016/j.jde.2017.04.032
   Impact Factor: 1.988
   Descriere. In acest articol se studiaza o clasa de probleme neliniare cu termen de reactie superliniar, potential nedefinit si conditie de tip Robin pe frontiera. Sunt demonstrate rezultate de existenta si proprietati de multiplicitate ale solutiilor, inclusive existenta unei infinitati de solutii. Se combina tehnici la interfata dintre analiza neliniara pura si cea aplicata iar demonstratiile folosesc metode rafinate de perturbare, de trunchiere si teorie Morse (grupuri critice).

7. N. Papageorgiou, **Vicentiu Radulescu,** Robin problems with indefinite and unbounded potential, resonant at $-\infty$, superlinear at ∞, *Tohoku Mathematical Journal* 69 (2017), 261-286
   doi: 10.2748/tmj/1498269626
   Impact Factor: 0.548
   Descriere. Se considera o clasa de probleme semiliniare cu conditie de tip Robin si neliniaritate cu comportament asimetric la cele doua capete ale axei reale. Rezonanta este permisa in origina in raport cu orice valoare proprie neprincipala a operatorului lui Laplace. Se obtin doua conditii suficiente pentru existenta a cel putin trei solutii netriviale. Lucrarea generalizeaza rezultate anterioare ale lui Recova si Rumbos.

8. K. Kefi, **Vicentiu Radulescu,** On a p(x)-biharmonic problem with singular weights, *Zeitschrift fuer angewandte Mathematik und Physik (ZAMP)* 68 (2017), 68:80
   DOI 10.1007/s00033-017-0827-3
   Impact Factor: 1.687
   Descriere. Se considera o clasa de probleme anisotrope cu mai multi exponenti variabili si conditie Navier pe frontiera. Problema este descrisa de un operator biarmonic de tip p(x)-Laplacian iar termenul neliniar contine termeni de tip convex-concav. Rezultatul principal stabileste existenta unei solutii netriviale cu energie negativa. Se stabileste si un rezultat de existenta in cazul perturbatiilor mici. Principiul variational al lui Ekeland joaca un rol important in demonstratiile principalelor rezultate din aceasta lucrare.

9. N. Papageorgiou, **Vicentiu Radulescu,** Asymmetric, noncoercive, superlinear *(p,2)*-equations, *Journal of Convex Analysis* 24 (2017), 769-793
   Impact Factor: 0.496
   Descriere. Se studiaza o clasa de probleme de tip Dirichlet descrise de suma dintre operatorul lui Laplace si operatorul p-Laplace. Termenul de reactie este asimetric, este superliniar in directia pozitiva si subliniar in directia negative. Ipotezele permit aparitia unor fenomene de rezonanta in raport cu orice valoare neprincipala a operatorului p-Laplace. Rezultatul principal stabileste existenta a cel putin trei solutii netriviale. Demonstratiile combina teoria punctului critic cu teoria Morse.

10. S. Liang, **Vicentiu Radulescu,** Infinitely many solutions for degenerate Kirchhoff-type Schrödinger-Choquard equations, *Electronic Journal of Differential Equations*, Vol. 2017 (2017), No. 230, pp. 1-17
    Impact Factor: 0.954
    Descriere. In acest articol se studiaza o clasa de ecuatii de tip Schrödinger-Choquard in varianta nelocala. Folosindu-se un rezultat recent de tip mountain pass al lui Kajikiya, se demonstreaza existenta unei familii infinite de solutii si se stabilesc mai multe proprietati calitative ale acestei familii. Problema este nedegenerata atunci cand termenul nelocal se anuleaza in origina.

11. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Pairs of positive solutions for resonant singular equations with the p-Laplacian, *Electronic Journal of Differential Equations*, Vol. 2017 (2017), No. 249, pp. 1-13
    Impact Factor: 0.954
    Descriere. Se studiaza o clasa de probleme quasiliniare cu conditie de tip Dirichlet pe frontiera. Termenul singular al ecuatiei este perturbat de o functie Caratheodory. Folosind metode variationale combinate cu tehnici de trunchiere si principia de comparative se demonstreaza existent a cel putin doua solutii netriviale.

12. B. Alleche, **Vicentiu Radulescu,** Further on set-valued equilibrium problems and applications to Browder variational inclusions, *Journal of Optimization Theory and Applications*, 175 (2017), 39-58
    DOI 10.1007/s10957-017-1169-1
    Impact Factor: 1.289
    Descriere. In acest articol sunt introduse unele notiuni de convexitate si semicontinuitate pentru aplicatii multivoce. Se studiaza apoi problema multivoca de echilibru si sunt stabilite mai multe conditii suficiente pentru existenta solutiilor. Rezultatele sunt aplicate pentru studierea incluziunilor variationale de tip Browder si sunt stabilite conditii mai generale pentru existenta solutiilor.

13. N. Papageorgiou, **Vicentiu Radulescu,** Positive solutions for parametric semilinear Robin problems with indefinite and unbounded potential, *Mathematica Scandinavica* 121 (2017), 263-292
    https://doi.org/10.7146/math.scand.a-96696
    Impact Factor: 0.635
    Descriere. Se studiaza o problema parametrica de tip Robin cu potential nedefinit si termen de reactie de tip Caratheodory care nu satisfice conditia Ambrosetti-Rabinowitz. Se stabileste o teorema generala de bifurcatie si se demonstreaza existenta solutiei minimale, precum si mai multe proprietati de monotonie si de continuitate ale acesteia. Demonstratiile combina tehnici variationale cu metode topologice si de monotonie.

14. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Nonhomogeneous hemivariational inequalities with indefinite potential and Robin boundary condition, *Journal of Optimization Theory and Applications*, 175 (2017), 293-323
    DOI: 10.1007/s10957-017-1173-5
    Impact Factor: 1.289

Descriere. Se studiaza o clasa de inegalitati hemivariationale cu potential nedefinit si conditie Robin. Termenul de reactie este diferentiala in sens Clarke al unei functionale local Lipschitz. Se demonstreaza existenta a cel putin trei solutii, doua avand semn constant si a treia fiind nodala. Demonstratiile combina teoria ne-neteda a punctului critic cu metoda directa de optimizare globala.

15. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Resonant semilinear Robin problems with a general potential, *Electronic Journal of Qualitative Theory of Differential Equations*, No. 70 (2017), 1-15

    https://doi.org/10.14232/ejqtde.2017.1.70

    Impact Factor: 0.926

    Descriere. In acest articol se studiaza o clasa de probleme semiliniare cu potential nemarginit si conditie Robin pe frontiera. Folosindu-se o varianta a metodei de reductie Lyapunov-Schmidt in combinatie cu metoda *local linking*, se demonstreaza ca problema are cel putin doua solutii netriviale. In plus, aplicandu-se teoria de regularitate a lui Lieberman, se arata ca aceste solutii sunt netede.

16. G. Li, **Vicentiu Radulescu**, D. Repovs, Q. Zhang, Nonhomogeneous Dirichlet problems without the Ambrosetti-Rabinowitz condition, *Topological Methods in Nonlinear Analysis (2017)*

    DOI: 10.12775/TMNA.2017.037

    Impact Factor: 0.667

    Descriere. In acest articol se analizeaza o clasa de probleme neliniare cu exponent variabil si conditie de tip Dirichlet pe frontiera. Se introduce o noua conditie de crestere pentru termenul neliniar si se stabilesc conexiuni cu conditia de compacitate a lui Cerami. Principalele rezultate stabilesc proprietati de existenta si de multiplicitate iar demonstratiile folosesc metode variationale si de teoria punctului critic.

17. **A. Golumbeamu, O.A. Ticleanu**, Elliptic curves differentiation with application to group signature scheme, *Electronic Journal of Differential Equations*, Vol. 2017(2017), No. 237, pp.1-21.

    URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu

    Impact Factor: 0.954

4. Lista Conferinte Invitate

[1] Vicentiu Radulescu, *Nonhomogeneous problems with singular weights***,** Fourth Conference on Recent Trends in Nonlinear Phenomena, University of Messina, 18-20 September 2017

http://www.sti.uniurb.it/servadei/ConferenceMessina2017

[2] Vicentiu Radulescu: *Maximum principle and Keller-Osserman condition revisited,* Faculty of Applied Mathematics, AGH University of Science and Technology, Krakow, 8 November 2017

http://wms.mat.agh.edu.pl/~afs/en/archiwum-2017.html

[3] Vicentiu Radulescu, *How much monotonicity is necessary in nonlinear PDEs?* Faculty of Mathematics and Applied Physics, Rzeszow University of Technology, Rzeszow, 17 November 2017

https://kan.prz.edu.pl/aktualnosci/zaproszenie-1.html

[4] Vicentiu Radulescu: *Nonlinear eigenvalue problems: old and new*, Chair of Optimization and Control, Jagiellonian University, Krakow, 23 November 2017

http://ww2.ii.uj.edu.pl/~migorski/links.html

5. Pozitii de Profesor Invitat

1. Vicentiu Radulescu: professor invitat la Univ. de Picardie "Jules Verne", Amiens, Franta (1-5 octombrie 2017)

2. Vicentiu Radulescu: profesor invitat la AGK University of Science and Technology, Krakow, Polonia (5-25 noiembrie 2017)

6. Carti si Capitole de Carti Publicate

1. **Vicentiu Radulescu**, G. Kassay: *Equilibrium Problems and Applications,* carte sub contract la Academic Press, Elsevier, Oxford (**Paperback ISBN:** 9780128110294). Data publicarii: 1 octombrie 2018:

https://www.elsevier.com/books/equilibrium-problems-and-applications/radulescu/978-0-12-811029-4

II. Etapa 2 – 2018

1. Rezumatul etapei

In cadrul acestei etape au fost realizate toate punctele asumate prin contract si au fost realizate si analize suplimentare. In acest mod au fost realizate studii ale problemelor critice si supercritice cu exponent variabil, ecuatii Schrodinger peste campuri fractionare precum si studiul problemelor critice si supercritice pentru ecuatii neliniare cu derivate partiale descrise de operatori cu unul sau mai multi exponenti variabili si al ecuatiilor Schrodinger peste campuri scalare fractionare:

1. Stabilirea unor formule de tip Pohozaev-Pucci-Serrin in cazul studiat
2. Studiul existentei solutiilor multiple in cazul supercritic
3. Studiul operatorilor poliharmonici in cazul descris

Conform rezultatelor obtinute au fost realizate toate obiectivele propuse, evidentiindu-se crearea unor premize pentru depasirea rezultatelor initial propuse si optimizari ale sistemelor de implementare.

2. Descrierea stiintifica si tehnica

Aceste studii au avut ca rezultat publicarea de articole stiintifice in Jurnale de prestigiu International, participari la Conferinte Internationale si pozitii de Profesor Invitat la Institutii de recunoastere internationala in domeniul studiat, dupa cum urmeaza in descrierea de mai jos:

3. Lista cu articolele publicate (selectie)

1. **Vicentiu Radulescu,** S. Saiedinezhad, A nonlinear eigenvalue problem with p(x)-growth and generalized Robin boundary value condition, *Communications on Pure and Applied Analysis* 17 (2018), 39-52.
2. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Nodal solutions for the Robin p-Laplacian plus an indefinite potential and a general reaction term, *Communications on Pure and Applied Analysis* 17 (2018), 231-241.
3. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Positive solutions for superdiffusive mixed problems, *Applied Mathematics Letters* 77 (2018), 87-93.
4. N. Papageorgiou, **Vicentiu Radulescu,** Multiplicity of solutions for nonlinear nonhomogeneous Robin problems, *Proceedings of the American Mathematical Society* 146 (2018), 601-611.
5. N. Papageorgiou, **Vicentiu Radulescu,** Semilinear Robin problems resonant at both zero and infinity, *Forum Mathematicum* 30 (2018), 237-251.

6. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Existence and multiplicity of solutions for resonant *(p,2)*-equations, *Advanced Nonlinear Studies* 18 (2018), 105-129.
7. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Resonant Robin problems driven by the p-Laplacian plus an indefinite potential, *Ann. Acad. Sci. Fennicae* 43 (2018), 483-508.
8. A. Bahrouni, **Vicentiu Radulescu,** D. Repovs, A weighted anisotropic variant of the Caffarelli-Kohn-Nirenberg inequality and applications, *Nonlinearity* 31 (2018), 1516-1534.
9. G. Li, **Vicentiu Radulescu,** D. Repovs, Q. Zhang, Nonhomogeneous Dirichlet problems without the Ambrosetti-Rabinowitz condition, *Topological Methods in Nonlinear Analysis* 51 (2018), 55-77.
10. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Periodic solutions for a class of evolution inclusions, *Computers and Mathematics with Applications* 75 (2018), 3047-3065.
11. N. Papageorgiou, **Vicentiu Radulescu,** Multiplicity of solutions for Robin problems with double resonance, *Annali della Scuola Normale Superiore di Pisa, Classe di Scienze, Serie V* XVIII (2018), 145-201.
12. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Positive solutions for nonlinear nonhomogeneous parametric Robin problems, *Forum Mathematicum* 30 (2018), 553-580.
13. A. Bahrouni, H. Ounaies, **Vicentiu Radulescu,** Compactly supported solutions of Schrödinger equations with small perturbation, *Applied Mathematics Letters* 84 (2018), 148-154.
14. **Vicentiu Radulescu,** X. Mingqi, B. Zhang, Nonlocal Kirchhoff diffusion problems: local existence and blow-up of solutions, *Nonlinearity* 31 (2018), 3228-3250.
15. K. Kefi, **Vicentiu Radulescu,** Small perturbations of nonlocal biharmonic problems with variable exponent and competing nonlinearities, *Rend. Lincei Mat. Appl.* 29 (2018), 439-463.
16. P. Pucci, **Vicentiu Radulescu,** The maximum principle with lack of monotonicity, *Electronic Journal of Qualitative Theory of Differential Equations* 2018, No. 58, 1-11.
17. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, *(p,2)*-equations symmetric at both zero and infinity, *Advances in Nonlinear Analysis* 7 (2018), 327-351.
18. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Positive solutions for nonvariational Robin problems, *Asymptotic Analysis* 108 (2018), 243-255.
19. N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Double-phase problems with reaction of arbitrary growth, *Zeitschrift fuer angewandte Mathematik und Physik (ZAMP)* 69 (2018), 69:108.
20. R. Alsaedi, **Vicentiu Radulescu,** Generalized biharmonic problems with variable exponent and Navier boundary condition, Two nonlinear days in Urbino 2017, *Electronic Journal of Differential Equations*, Conf. 25 (2018), pp. 27-37.
21. M. Cencelj, **Vicentiu Radulescu,** D. Repovs, Double phase problems with variable growth, *Nonlinear Analysis* 177 (2018), 270-287.
22. X. Mingqi, **Vicentiu Radulescu,** B. Zhang, Combined effects for fractional Schrödinger-Kirchhoff systems with critical nonlinearities, *ESAIM: COCV* 24 (2018), 1249-1273.

4. Lista Conferinte Invitate

1."Alcuni modelli matematici nelle scienze applicate: singolarità, frattali e fluidi non-newtoniani", Accademia delle Scienze dell'Umbria, 10 January 2018

2. “Small and high perturbations of nonhomegeneous elliptic problems”, Two Nonlinear Days in Perugia on the occasion of Patrizia Pucci's 65th birthday, University of Perugia, 11-12 January 2018

3. “Equilibrium problems and applications”, Fourth Conference on Mathematical Sciences and Applications, King Saud University, Riyadh, 11-12 April 2018

5. Raport Stiintific asupra cercetarii cu precizarea rezultatelor importante obtinute

Lucrarea 10: N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Periodic solutions for a class of evolution inclusions, *Computers and Mathematics with Applications* 75 (2018), 3047-3065.

We consider a periodic evolution inclusion defined on an evolution triple of spaces. The inclusion involves also a subdifferential term. We prove existence theorems for both the convex and the nonconvex problem, and we also produce extremal trajectories. Moreover, we show that every solution of the convex problem can be approximated uniformly by certain extremal trajectories (strong relaxation). We illustrate our results by examining a nonlinear parabolic control system.

Lucrarea 11: N. Papageorgiou, **Vicentiu Radulescu,** Multiplicity of solutions for Robin problems with double resonance, *Annali della Scuola Normale Superiore di Pisa, Classe di Scienze, Serie V* XVIII (2018), 145-201.

We consider Robin boundary value problems with a reaction exhibiting double resonance at ±1 with respect to any nonprincipal spectral interval. We prove several multiplicity theorems, producing nontrivial smooth solutions with sign information. We also prove an exact multiplicity theorem. We employ variational tools from critical point theory, together with truncation-perturbation techniques, flow invariance arguments and Morse theory (critical groups). We produce up to seven nontrivial smooth solutions all with sign information.

Lucrarea 17: N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, *(p,2)*-equations symmetric at both zero and infinity, *Advances in Nonlinear Analysis* 7 (2018), 327-351.

In this paper, we consider a class of nonlinear problems with unbalanced growth. More precisely, we develop the mathematical analysis of a $(p, 2)$-equation, that is, a nonlinear nonhomogeneous elliptic equation driven by the sum of a p-Laplacian and a Laplacian with $p > 2$. The reaction term is $(p-1)$-linear, but exhibits asymmetric behavior at infinity and at the origin. Using variational tools, together with truncation and comparison techniques and Morse theory, we prove two multiplicity theorems, one of them providing sign information for all the solutions (positive, negative, nodal) of the problem. The methods developed in this paper can be applied to wide classes of isotropic double-phase problems.

Lucrarea 19: N. Papageorgiou, **Vicentiu Radulescu,** D. Repovs, Double-phase problems with reaction of arbitrary growth, *Zeitschrift fuer angewandte Mathematik und Physik (ZAMP)* 69 (2018), 69:108.

We consider a parametric nonlinear nonhomogeneous elliptic equation, driven by the sum of two differential operators having different structure. The associated energy functional has unbalanced growth and we do not impose any global growth conditions to the reaction term, whose behavior is prescribed only near the origin. Using truncation and comparison techniques and Morse theory, we show that the problem has multiple solutions in the case of high perturbations. We also show that if a symmetry condition is imposed to the reaction term, then we can generate a sequence of distinct nodal solutions with smaller and smaller energies.

Lucrarea 21: M. Cencelj, **Vicentiu Radulescu,** D. Repovs, Double phase problems with variable growth, *Nonlinear Analysis* 177 (2018), 270-287.

We consider a class of double phase variational integrals driven by nonhomogeneous potentials. We study the associated Euler equation and we highlight the existence of two different Rayleigh quotients. One of them is in relationship with the existence of an infinite interval of eigenvalues while the second one is associated with the nonexistence of eigenvalues. The notion of eigenvalue is understood in the sense of pairs of nonlinear operators, as introduced by Fučík, Nečas, Souček, and Souček. The analysis developed in this paper extends the abstract framework corresponding to some standard cases associated to the $p(x)$-Laplace operator, the generalized mean curvature operator, or the capillarity differential operator with variable exponent. The results contained in this paper complement the

pioneering contributions of Marcellini, Mingione et al. in the field of variational integrals with unbalanced growth.

III. Etapa 3 – 2019

1. Rezumatul etapei

Etapa a treia a proiectului a avut in vedere realizarea tuturor punctelor asumate cat si concluzionarea studiului prin integrarea rezultatelor suplimentare obtinute in fiecare etapa. Suplimentar fata de clauzele asumate, in cadrul acestei etape, au fost obtinute rezultate in domeniul existentei solutiilor pentru probleme superliniare de tipul Kirchhoff in cazul operatorilor Laplace fractionari. In capitolul 8 al Raportului sunt descrise rezultatele obtinute precum si recunoasterea lor de catre comunitatea stiintifica internationala, prin prestigiul jurnalelor in care au fost publicate.

Rezultatele obtinute si sustinute de publicarea lor in jurnale de prestigiu international, indexate ISI, cu factor de impact, a avut un ecou in crearea de noi colaborari si deschiderea unei noi arii de cercetare interdisciplinara prin cooptarea de noi membri in cadrului Laboratorului de Cercetare Modele de Analiza Neliniara si Aplicatii – MANA (http://stiinte.ucv.ro/mana/).

Conform rezultatelor obtinute au fost realizate toate obiectivele propuse, evidentiindu-se crearea unor premize si concretizarea lor pentru depasirea rezultatelor initial propuse si optimizari ale sistemelor de implementare datorate DataRoom-ului existent in Laboratorul de Cercetare MANA (http://stiinte.ucv.ro/mana/) si cooptarea in cadrul echipei a noi colaboratori din tara si strainatate.

2. Descrierea stiintifica si tehnica

Studiile efectuate au fost publicate si au fost efectuate prezentari la conferintele invitate in cadrul scolilor de cercetare de renume mondial, in matematica aplicata, precum si prin obtinerea de invitatii ca si editor de volume ale studiilor efectuate in colaborare cu cercetatori de prestigiu in domeniu, rezultate prezentate in cele ce urmeaza.

3. Lista cu articolele publicate (selectie)

[1] N.S. Papageorgiou, **V.D. Radulescu**, D.D. Repovs, Asymmetric Robin problems with indefinite potential and concave terms, *Adv. Nonlinear Stud.* **19** (2019), 69-87.
https://doi.org/10.1515/ans-2018-2022
2018 ISI Impact Factor: 1.65

[2] A. Bahrouni, H. Ounaies, **V.D. Radulescu**, Bound state solutions of sublinear Schrodinger equations with lack of compactness, *RACSAM* **113** (2019), 1191-1210.
https://doi.org/10.1007/s13398-018-0541-9
2018 ISI Impact Factor: 1.028

[3] N.S. Papageorgiou, **V.D. Radulescu**, D.D. Repovs, Periodic solutions for implicit evolution inclusions, *Evolution Equations and Control Theory* **8** (2019), 621-631.
doi:10.3934/eect.2019029
2018 ISI Impact Factor: 1.049

[4] N.S. Papageorgiou, **V.D. Radulescu**, D.D. Repovs, Positive solutions for nonlinear parametric singular Dirichlet problems, *Bulletin of Mathematical Sciences* **9** (2019), 1950011 (21 pages).
DOI: 10.1142/S1664360719500115
2018 ISI Impact Factor: 1.714

[5] Z. Binlin, **V.D. Radulescu**, L. Wang, Existence results for Kirchhoff-type superlinear problems involving the fractional Laplacian, *Proceedings of the Royal Society of Edinburgh* **149** (2019), 1061-1081.
DOI:10.1017/prm.2018.105
2018 ISI Impact Factor: 1.045

4. Lista Conferinte Invitate

[1] V. Radulescu, Anisotropic problems: qualitative results, Potentiel et Probabilités, Institute of Mathematics of the Romanian Academy, Bucharest, 24-25 January 2019

[2] V. Radulescu, Anisotropic double phase problems and perspectives, King Saud University, Riyadh, 3 April 2019

[3] V. Radulescu, Nonlinear problems with unbalanced growth: isotropic and anisotropic
models, Stochastic Analysis and Related Topics, University of Bucharest, 6-9 May 2019

[4] V. Radulescu, Problèmes à double phase et croissance variable, LAMFA, Universite de Picardié Jules Verne, Amiens, 20 May 2019

[5] V. Radulescu, *Double phase problems with variable growth*, BiUrb Recent advances in variational methods, Università di Urbino Carlo Bo, 29 May 2019

[6] V. Radulescu, *Singular phenomena in nonlinear elliptic equations*, BiUrb Recent advances in variational methods, Università di Urbino Carlo Bo, 30 May 2019

[7] V. Radulescu, Problems with variable exponent, Elsevier-JMAA Conference on Nonlinear Analysis at AGH-UST, Krakow, 11-12 October 2019

5. Pozitii de Profesor Invitat

1. V. Radulescu, visiting professor at King Saud University, Riyadh, Saudi Arabia (1-6 April 2019)
2. V. Radulescu, visiting professor at Universite de Picardié Jules Verne, Amiens, France (19-25 May 2019)
3. V. Radulescu, visiting professor at Università di Urbino Carlo Bo, Urbino, Italy (27 May – 1 June 2019)
4. V. Radulescu, visiting professor at AGH University of Science and Technology, Krakow, Poland (30 September – 23 October 2019)
5. V. Radulescu, visiting professor at Central South University, Changsha, China (19 November – 20 December 2019)

6. Carti si Capitole de Carti Publicate

1. N.Papageorgiou, **V.D. Radulescu,** D. Repovs, *Nonlinear Analysis – Theory and Methods*, Springer Monographs in Mathematics, Springer, Cham, 2019, 577 pp.

eBook ISBN: 978-3-030-03430-6
DOI: 10.1007/978-3-030-03430-6
Hardcover ISBN: 978-3-030-03429-0
Series ISSN: 1439-7382

7. Editor de Volum

1. P. Pucci, V.D. Radulescu, of the Special Issue *Progress in Nonlinear Kirchhoff Problems*, *Nonlinear Analysis*, vol. 186, pp. 1-258, September 2019

8. Raport Stiintific asupra cercetarii cu precizarea a trei rezultate importante obtinute

The research activity developed in the papers published in 2019 in the framework of the Research Project 23/2017 (code PN-III-P4-ID-PCE-2016-0130) focused on the qualitative, quantitative and numerical analysis of some classes of anisotropic differential systems and applications. In this framework, we have been constantly interested in some of the open problems raised by G. Mingione in 2016 concerning the analysis of *double-phase anisotropic problems*. We have considered both differential and nonlocal systems. The first class of nonlinear problems is raised by (isotropic or anisotropic) differential systems, while the second class is raised by fractional Laplace-type operators or Kirchhoff integro-differential operators. The best three results obtained in the papers reported for 2019 are the following.

Result No. 1. In paper [1] we have been concerned with a class of asymmetric differential systems with Robin boundary condition. The source term is affected by a linear perturbation and in the reaction we distinguish the competition effects of a concave term and of an asymptotically linear term which is resonant in the negative direction. The main results of this paper establish the existence of at least four (respectively, five) solutions. These solutions are nontrivial and smooth and they appear in the case of *low perturbations*. The proofs combine refined analytic, variational and topological methods, including Morse theory (critical groups), truncation methods and perturbation techniques. The results are new and the methods are quite general; we expect that they will be extended to wide classes of isotropic or anisotropic nonlinear problems.

Result No. 2. Paper [4] is devoted to the study of a class of quasilinear systems with Dirichlet boundary condition. The feature of this paper is the presence of a nonlinear singular parametric term in the reaction. We also point out the perturbation effects created by a nonlinearity with almost linear growth at infinity. At the same time, the

problem is uniformly nonresonant with respect to the principal eigenvalue of the differential operator which controls the problem. A basic result established in this paper describes exhaustively a bifurcation-type property with respect to all the values of a positive parameter. Namely, the problem has at least two positive solutions for values of the parameter less than the critical value, at least one solution in the critical case, and no solutions for supercritical values of the parameter. The main result is a connected with pioneering contributions of Crandall, Rabinowitz, Tartar, Coclite, Hirano in the study of nonlinear *singular* problems. Some of the main abstract tools found in this paper have been introduced in our recent monograph

N.Papageorgiou, **V.D. Radulescu,** D. Repovs, *Nonlinear Analysis – Theory and Methods*, Springer Monographs in Mathematics, Springer, Cham, 2019, 577 pp.

Result No. 3. Paper [5] deals with the qualitative analysis of a superlinear Kirchhoff system driven by a nonlocal integral-differential operator and by the fractional Laplace operator. By computing the critical groups at zero and at infinity, we obtain the existence of nontrivial solutions via Morse theory. A feature of this paper is that the reaction is very general and it does not satisfy the standard Ambrosetti-Rabinowitz condition. That is why, many classes of nonlinear terms are allowed to satisfy the general assumptions of this paper. The main result is the first existence property obtained in the case of Kirchhoff-type Laplacian problems. At the same time, the paper extends to the case of Kirchhoff problems several results developed in our monograph

G. Molica Bisci, **V.D. Radulescu**, R. Servadei, *Variational Methods for Nonlocal Fractional Problems*, Encyclopedia of Mathematics and its Applications, vol. 162, Cambridge University Press, Cambridge, 2016.

Director Proiect

Prof. Univ. Dr. Vicentiu Radulescu